

## A NUMERICAL SIMULATION FOR A DETERMINISTIC COMPARTMENTAL MATHEMATICAL MODEL OF HIV/AIDS DISEASE PANDEMIC WITH EFFECT OF PUBLIC AWARENESS/CAMPAIGN

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### Abstract

*In this work, an earlier deterministic mathematical model of HIV/AIDS is revisited and numerical solutions obtained using Euler's numerical method. Using hypothetical values for the parameters, a program was written in VISUAL BASIC programming language to generate series for the system of difference equations from the output of the program. Simulation results were presented in form of graphs. Finally, results were compared with the earlier analysis which was carried out using a modified version of Bellman and Cooke's theory. It was found that both results agree.*

**Keywords:** Deterministic, HIV/AIDS, model, simulation

### Introduction

AIDS is an acronym for Acquired Immunodeficiency Syndrome. The disease is not hereditary but develops after having contact with a disease causing agent called Human Immunodeficiency Virus (HIV).

It is characterized by weakening of the immune system. HIV destroys a certain kind of blood cell (CD4+ T cells) which is crucial to the normal function of the human immune system. In fact, loss of these cells in people with HIV is an extremely powerful predictor of the development of AIDS. Studies of thousands of people have revealed that most people infected with HIV carry the virus for years before enough damage is done to the immune system for AIDS to develop. However, sensitive tests have shown a strong connection between the amount of HIV in the blood and the decline in CD4+ T cells and the development of AIDS. Reducing the amount of virus in the body with anti-retroviral therapies can dramatically slow the destruction of a person's immune system.

It must be emphasized that there are no specific symptoms of HIV infection and infected persons may remain asymptomatic for years due to the long latency period associated with the infection. Clinical symptoms usually subside and infected patients may enter long latency periods of up to 10 years, during which the virus continue to replicate and viral load increases while the patient's CD<sub>4</sub> count decreases. Eventually the body's immune system is overwhelmed and the patient manifests with symptoms and signs of infections called 'opportunistic infections'. For example *pneumocystis carinii pneumonia*, disseminated *tuberculosis*, Oesophageal, and central nervous system *mycosis* and gastrointestinal infections causing chronic diarrhea. The presence of a sexually transmitted disease in a patient prompts the suspicion of HIV infection. This is because HIV is mainly transmitted sexually and patients with STDs like syphilis gonorrhoea, lymphogranuloma venereum and genital herpes have an increased risk of developing HIV infection because they share the same

sexual behaviour and the presence of genital ulcers facilitate viral entry.(Kaidal,2007).

Frequently observed symptoms associated with HIV/AIDS include unexplained prolonged fevers for more than one month, weight loss, skin lesion e.g. *seborrhoeic dermatitis*, *prougoi* fungal nail infections, recurrent mouth ulcers, oral thrash, oral hairy *leukoplakia*, persistent cough generalized *lymphadenopathy* purulent abscesses in the muscle. A diagnosis of pulmonary or disseminated tuberculosis, Herpes Zoster, Kaposi sarcoma and CNS Cryptococcus also raise a suspicion of HIV infection.

HIV is spread by sexual contact with an infected person, by sharing needles and/or syringes (primarily for drug injection) with someone who is infected, or, less commonly (and now very rarely in countries where blood is screened for HIV antibodies), through transfusions of infected blood or blood clotting factors. Babies born to HIV-infected women may become infected before or during birth or through breast-feeding after birth.

### Literature review

For most models , some kinds of computer program were used to estimate the parameters of the model and obtain the results. These kinds of programs are more of tailor-made programs designed to solve only the problem at hand and might not be easily applicable for use in other contexts. In this subsection, we review some of those programs that have been developed for generic use. The programs/software falling in this category includes the following.

- i. The EpiModel developed by the WHO Global Programme on AIDS (GPA) for short-term forecasting of AIDS. The model has been described in Chin *et al.* (1989), While EpiModel does not produce

an automatic projection of future prevalence, it does make it easier for users to prepare such an estimate.” (Stover, 1997:12-13)

- ii. AIDSTECH and AIDSproj (an Excel spreadsheet software) developed by Stover and incorporates AIDSTECH and the methodology in EpiModel as described in Chin and Lwanga (1991) used for estimating HIV prevalence and for subsequently calculating AIDS cases and AIDS deaths. AIDSProj has similar features as EpiModel. One key difference between the two is that AIDSproj uses as input the prevalence of HIV as a percentage of all adults rather than the number of infections (as done in EpiModel). In this way, the projection of future prevalence does not rely on a population projection (since it is expressed as a percentage) (Stover, 1997).
- iii. AIM also developed by Stover and used to illustrate the future consequences of the epidemic.included as a module in integrated software, Spectrum, developed by Futures Group International.The iwgAIDS (Interagency Working Group on AIDS) model is a complex simulation model of the spread of HIV through a population as a result of the behavior of various population subgroups (Stanley *et al.*, 1989). The US Census Bureau approach to projecting HIV prevalence is based on this.
- iv. The Doyle model, ASSA500 and ASSA600 models are interrelated and were all developed in South Africa. The Doyle model was produced by Metropolitan Life and

was later adapted by the Actuarial Society of South Africa (ASSA) to produce ASSA500 and ASSA600 spreadsheet models. At the core of the ASSA models, The model has been widely used in South Africa

by various modellers and interest groups.

**Mathematical model**

We revisit the Mathematical Model equations of HIV/AIDS disease pandemic presented by Enagi (2008).

$$P = S + R + L + I$$

$$S' = (\beta - \mu - \gamma)S(t) + \beta R(t) + \theta \beta L(t) - \alpha S(t)[L(t) + I(t)] \quad (2.1)$$

$$R' = [\gamma S(t) - \mu R(t)] \quad (2.2)$$

$$L' = [(1 - \theta)\beta - \mu - \tau]L(t) + \alpha S(t)[L(t) + I(t)] \quad (2.3)$$

$$I' = (\beta - \mu - \delta)I(t) + \tau L(t) \quad (2.4)$$

with the parameters given by

$\beta$  = natural birth rate for the population

$\mu$  = natural death rate for the population

$\delta$  = death modulus due to infection

$\alpha$  = rate of contracting the HIV virus

$\gamma$  = rate of removal of the susceptible into the removed class due to public campaign

$\tau$  = rate of flow from the latent class into the infected class

$\theta$  = the proportion of the off-spring of the latent which are virus free at birth  $0 \leq \theta \leq 1$

t = time

Using Euler’s numerical method with step size h=1, we obtained the following Difference equations from (2.1) – (2.4)

$$S_{n+1} = S_n + S'_n$$

where

$$S'_n = (\beta - \mu - \gamma)S_n(t) + \beta R_n(t) + \theta\beta L_n(t) - \alpha S_n(t)[L_n(t) - I_n(t)]$$

hence

$$S_{n+1} = S_n + (\beta - \mu - \gamma)S_n(t) + \beta R_n(t) + \theta\beta L_n(t) - \alpha S_n(t)[L_n(t) - I_n(t)] \quad (2.5)$$

$$R_{n+1} = R_n + R'_n$$

where

$$R'_n = [\gamma S_n(t) - \mu R_n(t)]$$

hence

$$R_{n+1} = R_n + [\gamma S_n(t) - \mu R_n(t)] \quad (2.6)$$

$$L_{n+1} = L_n + L'_n$$

where

$$L'_n = [(1 - \theta)\beta - \mu - \tau]L_n(t) + \alpha S_n(t)[L_n(t) + I_n(t)]$$

hence

$$L_{n+1} = L_n + [(1 - \theta)\beta - \mu - \tau]L_n(t) + \alpha S_n(t)[L_n(t) + I_n(t)] \quad (2.7)$$

$$I_{n+1} = I_n + I'_n$$

where

$$I'_n = (\beta - \mu - \delta)I_n(t) + \tau L_n(t)$$

hence

$$I_{n+1} = I_n + (\beta - \mu - \delta)I_n(t) + \tau L_n(t) \quad (2.8)$$

### The programme

we write a program in VISUAL BASIC programming language which generates series for the above system of difference equations.

Project: "myProject"

Revision Number: 1.0.0

Revision Date: 05/07/2010 4:47:08 AM

File Name: "myProject.exe"

Location: C:\VB Projects\VB Projects 2010\myProject\myProject.vbp

Module1.bas

(Declarations)

Sub Main

frmMain.frm

(Declarations)

Sub Form\_Load

Sub cmdNew\_Click

Sub InitialiseVariables

Sub mnuCopySn\_Click

Sub mnuCopyN\_Click

Sub mnuCopyL\_Click

Sub mnuCopyR\_Click

Sub muCopyIn\_Click

Sub mnuData\_Click

Sub cmdCompute\_Click

Sub mnuFileNew\_Click

Sub mnuHelpAbout\_Click

Sub mnuN\_Click

Sub mnuViewStatusBar\_Click

Sub ClearTextBoxes

Sub ClearListView

Sub mnuFileExit\_Click

Sub Form\_Unload

frmOptions.frm  
(Declarations)  
Sub cmdApply\_Click  
Sub cmdCancel\_Click  
Sub cmdOK\_Click  
Sub Form\_Load

Objects:

Microsoft Common Dialog Control 6.0 (SP6)  
Path: C:\WINDOWS\system32\comdlg32.ocx  
Microsoft Windows Common Controls 6.0  
(SP6)  
Path:  
C:\WINDOWS\system32\MSCOMCTL.OCX

-----  
References:  
OLE Automation  
Path: C:\WINDOWS\system32\stdole2.tlb  
-----

-----  
Summary:  
Blank Lines           13  
Lines of Code         114  
Lines of Comments     6  
-----

Total Lines (Comments & Code) 120

-----  
'/\*\*\*\*\*\*  
'// File Name: frmMain.frm  
'// File Size: 23.4 KB  
'// File Date: 7/5/10 4:45:12 AM  
'// Printed On: Mon. July 5, 2010 4:47:35 AM  
'/\*\*\*\*\*\*

Private Sub Form\_Load()  
N = GetSetting(App.Title, "Settings", "N", N)  
A = GetSetting(App.Title, "Settings", "A", A)  
ClearTextboxes  
End Sub  
Text(6).Text = 0.025  
Text(7).Text = 60  
Text(8).Text = 20  
Text(9).Text = 10  
Text(10).Text = 10

Private Sub cmdNew\_Click()  
'clear textboxes  
ClearTextboxes  
ClearListView  
End Sub  
AP = Text(0).Text  
BT = Text(1).Text  
TT = Text(2).Text  
GM = Text(3).Text  
MU = Text(4).Text  
DT = Text(5).Text

Sub InitialiseVariables()  
Text(0).Text = 0.01  
Text(1).Text = 0.02  
Text(2).Text = 0.5  
Text(3).Text = 0.1  
Text(4).Text = 0.015  
Text(5).Text = 0.01  
TA = Text(6).Text  
S(0) = Text(7).Text  
R(0) = Text(8).Text  
L(0) = Text(9).Text  
i(0) = Text(10).Text  
End Sub

```

Private Sub mnuData_Click()
InitialiseVariables
End Sub

Private Sub cmdCompute_Click()
On Error Resume Next

Redim S(N), R(N), L(N), i(N)

ClearListView

For x = 0 To 10
If Text(x).Text = "" Then
MsgBox "Check your Input Parameters",
vbExclamation, "Missing Value"
Exit Sub
End If
Next

For K = 0 To N

Set lst = ListView.ListItems.Add(, , K, p, p)
lst.SubItems(1) = Format(S(K), "0")
lst.SubItems(2) = Format(R(K), "0")
lst.SubItems(3) = Format(L(K), "0")

lst.SubItems(4) = Format(i(K), "0")

S(K + 1) = S(K) + (BT - MU - GM) * S(K) +
BT * R(K) + TT * BT * L(K) - AP * (L(K) +
1)
R(K + 1) = R(K) + GM * S(K) - MU * R(K)
L(K + 1) = L(K) + ((1 - TT) * BT - MU - TA)
* L(K) + AP * S(K) * (L(K) + 1)
i(K + 1) = i(K) + (BT - MU - DT) * i(K) + TA
* L(K)

Next K
End Sub

Sub ClearTextBoxes()
'clears all textboxes
For i = 0 To 10
Text(i).Text = ""
Next
End Sub

Sub ClearListView()
On Error Resume Next
Dim i As Integer
For i = 1 To ListView.ListItems.Count
ListView.ListItems.Remove 1
Next i
End Sub

```

## Results of simulation

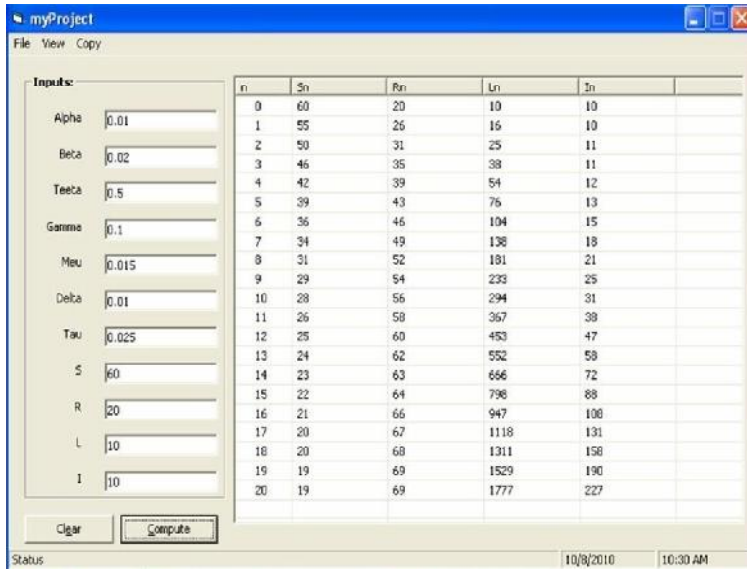


Figure 1 Showing the program inter phase  
The graphs given below where obtained from the output of the program using Microsoft excel spread sheet package.

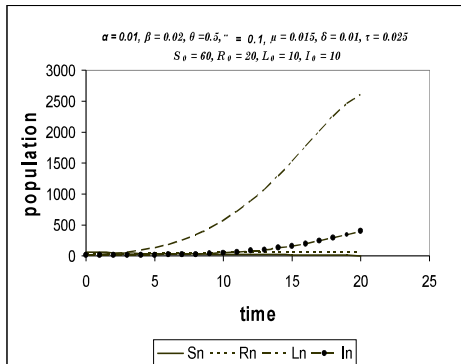


Figure 2 Showing graphical profile of each class for  $\alpha = 0.01, \beta = 0.02, \theta = 0.5, \gamma = 0.1, \mu = 0.015, \delta = 0.01, \tau = 0.025, S_0 = 60, R_0 = 20, L_0 = 10 \& I_0 = 10$

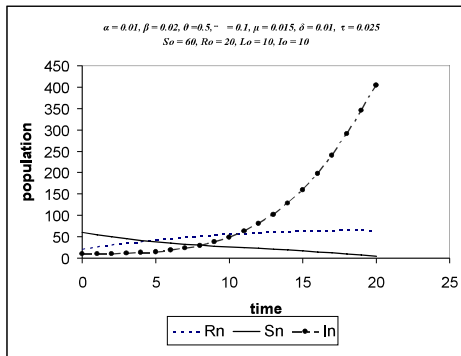


Figure 3 Showing a closer view of  $S_n$ ,  $R_n$  and  $L_n$  for  $\alpha = 0.01$ ,  $\beta = 0.02$ ,  $\theta = 0.5$ ,  $\gamma = 0.1$ ,  $\mu = 0.015$ ,  $\delta = 0.01$ ,  $\tau = 0.025$ ,  $S_0 = 60$ ,  $R_0 = 20$ ,  $L_0 = 10$  &  $I_0 = 10$

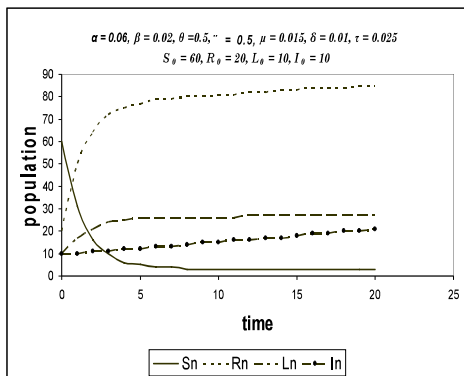


Figure 4 Showing graphical profile of each class for  $\alpha = 0.06$ ,  $\beta = 0.02$ ,  $\theta = 0.5$ ,  $\gamma = 0.5$ ,  $\mu = 0.015$ ,  $\delta = 0.01$ ,  $\tau = 0.025$ ,  $S_0 = 60$ ,  $R_0 = 20$ ,  $L_0 = 10$ ,  $I_0 = 10$

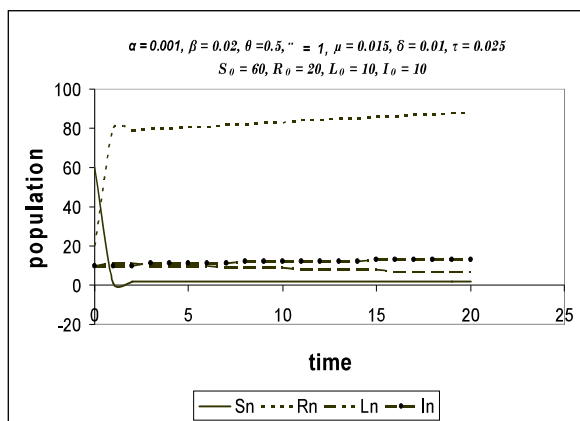


Figure 5 Showing graphical profile of each class for  $\alpha = 0.001$ ,  $\beta = 0.02$ ,  $\theta = 0.5$ ,  $\gamma = 1$ ,  $\mu = 0.015$ ,  $\delta = 0.01$ ,  $\tau = 0.025$ ,  $S_0 = 60$ ,  $R_0 = 20$ ,  $L_0 = 10$ ,  $I_0 = 10$

Since from our assumptions the population has constant natural Birth and Death rates  $\beta$  and  $\mu$  respectively, we set  $\beta = 0.02$  and  $\mu = 0.015$ . We also assumed that half of the offspring of the latent class are virus free at birth therefore  $\theta = 0.5$  we then vary the measure of effectiveness of public awareness and campaign ( $\gamma$ ) which resulted in changed of behaviour of members of susceptible class. Increase in ( $\gamma$ ) results in decrease in contraction rate ( $\alpha$ ) and vice versa. The movement rate from the Latent class to the

infected class ( $\square$ ) and the death rate with respect to infection ( $\delta$ ) were kept constant because they depend on  $\alpha$  they increase as  $\alpha$  increases and decrease as it decreases, hence we set  $\square = 0.025$  and  $\delta = 0.01$ . We take a sample population of 100, 60 of them are susceptible, 20 Removed class, 10 Latent and 10 infected as the initial populations.

### Discussion of results

Figure 2 gives graphical profiles of each class when  $\alpha = 0.01$  and  $\gamma = 0.1$ , with The initial



population of 100.  $S_0 = 60$ ,  $R_0 = 20$ ,  $L_0 = 10$  and  $I_0 = 10$ . We observed that the population of the Latent class increases rapidly, thus giving an unstable population.

Figure 3 gives a closer look at  $S_n$ ,  $R_n$  and  $I_n$ . We observed that the Susceptible class decreases gradually while the Infected class increases exponentially, this is because  $\gamma$  is low and  $\alpha$  is high. Figure 4 gives graphical profiles of each class when  $\alpha = 0.06$  and  $\gamma = 0.5$ , We observed that  $S_n$  decrease rapidly and became stable at  $n = 6$ , while  $R_n$  increases rapidly and also became stable at  $n = 6$ . This gives a stable population at  $n = 6$ .

Figure 5 gives gives graphical of each class when  $\alpha = 0.001$  and  $\gamma = 1.0$ .  $S_n$  decreased very rapidly and became stable at  $n = 2$ . There was a slight increase in  $L_n$  and a slight decrease in  $I_n$ . Thus a stable population at  $n = 2$ .

### Conclusion

The results from numerical simulations above agrees with that of Enagi (2008). It was observed that the population became stable at  $n = 6$  from figure 4.3 and at  $n = 2$  from figure 4.4, this shows that the higher the measure of effectiveness of public awareness/campaign ( $\gamma$ ) the lower the contraction rate ( $\alpha$ ) and the shorter the time before the stability of the population.

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