MATHEMATICAL SOLUTIONS FOR HEPATITIS B VIRUS INFECTION IN NIGERIA

S. Abdulrahman, N. I. Akinwande and U. Y. Abubakar

Department of Mathematics and Statistics, Federal University of Technology, Minna

O. B. Awojoyogbe
Department of Physics, Federal University of Technology, Minna
E-mail: sirajo.abdul@futminna.edu.ng

Abstract

This paper is an analysis of the transmission dynamics and control of hepatitis B virus (HBV) infection in Nigeria using mathematical model. \( R_e \) was obtained as the effective basic reproduction number, and its values computed using 6 different control strategies. Result shows that with 25 years waning rate of vaccine, HBV cannot be controlled through vaccination of new births or of susceptible individuals before being sexually active at any coverage rate. Two feasible solutions to the country’s HBV infection menace were obtained. The first is condom usage of 20% by sexually active individuals with 66% vaccination coverage of new births and 20% vaccination coverage each of sexually active and yet to be sexually active susceptible individuals. The second feasible solution is 20% condom usage with 30% vaccination coverage of sexually active susceptible individuals.

Keywords: Hepatitis B virus, Effective basic reproduction number, Vaccination coverage

Introduction

Hepatitis (plural Hepatitis) is a general term that means injury to the liver characterized by the presence of inflammatory cells in the tissue of the organ (liver). Hepatitis B is a disease caused by hepatitis B virus (HBV). This disease reduces the liver’s ability to perform life-preserving functions, including filtering harmful infectious agents from the blood, storing blood sugar and converting it to usable energy forms, and producing many proteins necessary for life.

Hepatitis B is fifty to one hundred times more infectious than HIV (WHO, 2009). It has caused epidemics in part of Asia and Africa, and it is endemic in China (Williams, 2006) and Nigeria (Adeoye, 2010). About a third of the world’s population, more than two billion peoples have been infected with hepatitis B virus at some stage in their life time. Of these, about 360 million people remain chronically infected carriers of the disease, most of whom are unaware of their HBV status (Long, Qi and Huang, 2008).

Transmission of hepatitis B virus results from exposure to infectious blood or body fluids containing blood. Possible forms of transmissions include (but are not limited to) unprotected sexual contact, blood transfusions, re-use of contaminated needles and syringes, and vertical transmission from mother to child during child birth (Wiah, Dontwi, & Adetunde, 2011).

Infection with the HBV has been a major public health problem. This has two phases: Acute and Chronic. The Acute phase causes liver inflammation, vomiting, and jaundice in which the individual is infectious. Chronic hepatitis B is an infection with hepatitis B virus that last longer than six months. Once the infection becomes chronic, it may never be cured completely, and may eventually cause liver cirrhosis and hepatocellular carcinoma (HCC) \([5,7]\). HBV causes approximately 600,000 deaths each year world-wide. Moreover, 10% of people infected with HIV (approximately four million people world-wide) are co-infected with HBV (Hoofnagle, Doo, Liang, Fleisher and Lok, 2007).

In order to find an efficient way to control an infection, it is of great importance to establish its transmission dynamics. One main goal of mathematical epidemiology is to understand how to control and eradicate diseases (Dahari, Shudo, Ribeiro and Perelson, 2009). Mathematical models are used extensively in the study of ecological and epidemiological phenomena (Ma, and Ma, 2006). They are particularly helpful as experimental tools with which to evaluate and compare control procedures and preventive strategies, and to investigate the relative effects of various sociological, biological and environmental factors on the spread of diseases. This is so because they can help.
in figuring out decisions that are of significance importance on the outcomes and provide comprehensive examinations that enter into decisions in a way that human reasoning and debate cannot.

During the last two and a half decades, (Kaplan & Brandeau, 1994) have designed mathematical models to evaluate the effect of public health programs and provided long-term predictions regarding HBV prevalence and control in various region. Nigeria, the Africa’s most populous country is classified among the group of countries highly endemic for HBV infection with about 20 million (13.14%) people infected. Mathematical solutions to the pandemic situation in Nigeria have not been address to the best of our knowledge.

Materials and methods
Model variables and parameters
The total population is compartmentalized into 8 epidemiological classes. The model applied has the following variables and parameters.

\[ \begin{align*}
S_U(t) & \quad \text{Susceptible individuals under 15 years of age at time } t \\
S_F(t) & \quad \text{Susceptible individuals at or above 15 years of age at time } t \\
V(t) & \quad \text{Vaccinated individuals at time } t \\
A_U(t) & \quad \text{Acutely infected individuals under 15 years of age at time } t \\
A_F(t) & \quad \text{Acutely infected individuals at or above 15 years of age at time } t \\
C_U(t) & \quad \text{Chronically infected individuals under 15 years of age at time } t \\
C_F(t) & \quad \text{Chronically infected individuals at or above 15 years of age at time } t \\
R(t) & \quad \text{Removed individuals due to recovery from infection at time } t \\
b & \quad \text{Per capital birth rate of humans} \\
\mu & \quad \text{Per capital natural death rate of humans} \\
\delta_A & \quad \text{HBV-induced death rate by } A_U \text{ and } A_F \\
\delta_C & \quad \text{HBV-induced death rate by } C_F \\
c & \quad \text{Average total sexual contacts} \\
p & \quad \text{HBV-Sexual transmission risk rate and therefore } pc \text{ is the effective sexual contact rate} \\
\eta & \quad \text{Modification parameter associated with reduced sexual transmission rate by chronic individuals} \\
\varepsilon_c & \quad \text{Condom efficacy} \\
\tau_c & \quad \text{Condom usage rate and therefore } \varepsilon_c \tau_c \text{ is the effective condom usage rate} \\
\varepsilon_p & \quad \text{Vaccine efficacy} \\
\tau_b & \quad \text{Vaccination coverage rate at birth and therefore } \rho_b = \varepsilon_p \tau_b \text{ is the effective vaccination rate at birth} \\
\tau_U & \quad \text{Vaccination coverage rate not at birth for } S_U \text{ and therefore } \rho_U = \varepsilon_p \tau_U \text{ is the effective vaccination rate for } S_U \text{ but not at birth} \\
\tau_F & \quad \text{Vaccination coverage rate for } S_F \text{ and therefore } \rho_F = \varepsilon_p \tau_F \text{ is the effective vaccination rate for } S_F \\
\omega & \quad \text{Loss (waning) rate of vaccination immunity} \\
\theta & \quad \text{Proportion of HBV- positive birth} \\
\phi & \quad \text{Modification parameter associated with reduced HBV – positive birth by } C_F \\
\sigma_S & \quad \text{Rate of moving from } S_U \text{ to } S_F
\end{align*} \]
The following assumptions are taken into account in the construction of the model.

1. There is homogeneous mixing of the population, where all people are equally likely to be infected by the infectious individuals in case of contact;
2. Public enlightenment (PE) is given to all classes and the higher the PE on HBV immunization and condom usage, the higher the compliance respectively;
3. Individuals in $S_U$, $A_U$ and $C_U$ classes are not yet sexually active, while those in $S_F$, $A_F$ and $C_F$ are sexually active.
4. By vaccination coverage we assumed the complete three dose of HBV vaccine.

The $S_U$ population are generated from daily recruitment of HBV uninfected individuals through birth given by
\[ bN(1 - \rho_b) - b\theta(A_F + \phi C_F)(1 - \rho_b) \]
where $\theta, 0 < \theta < 1$, is the fraction of the new birth that are born with HB virus into $A_U$ class (vertical transmission), as in Akinwande (2005). $\phi, 0 < \phi < 1$, is the modification parameter associated with reduced infectivity of $C_F$ individuals as in Brauer and Castillo-Chavez (2001) and $\rho_b, 0 \leq \rho_b < 1$, is the effective immunization rate (i.e. the product of vaccine efficacy and coverage at birth, $e_p \times \tau_b$).

The $S_F$ population are generated from $S_U$ and $V$ classes at the rates $\sigma_S$ and $\omega$ respectively, where $\sigma_S$ is the progression rate from $S_U$ to $S_F$ and $\omega$ is the rate of waning of vaccine efficacy. They acquired infection and move to the $A_F$ class via sexual transmission from individuals in the $A_F$ and $C_F$ compartments, given by
\[ \frac{\beta(A_F + \eta C_F)(1 - e_c \tau_c)}{N} \]

The parameter $\beta$ is the effective sexual contact rate (i.e. the product of the average total sexual contacts, $c$ and the probability of HB virus transmission, $p$). $\eta$ is the modification parameter associated with reduced sexual transmission rate by chronic individuals, as in Garba and Gumel (2010). The term $(1 - e_c \tau_c)$ reflect the impact of condom usage (efficacy and compliance) on sexual transmission, which is as a result of public enlightenment, $0 < e_c < 1$, and $0 < \tau_c < 1$.

The proportions $\rho_U = e_p \times \tau_U$ and $\rho_F = e_p \times \tau_F$ of $S_U$ and $S_F$ respectively are assumed to be vaccinated with 25 years-long immunity (i.e. $\omega = 0.04$) and therefore leave the Susceptible classes to the Vaccinated class as in Scherer and Mclean (2002), where $e_p$, $\tau_U$ and $\tau_F$ are rates of efficacy of vaccination, $S_U$ and $S_F$ compliance to vaccination respectively. Individuals in $A_U$, $A_F$ and $C_F$ classes acquired recovery from HBV infection with life.
immunity at the rate $\gamma_U = \sigma_A (1 - \varphi_U)$, $\gamma_F = \sigma_A (1 - \varphi_F)$ and $\gamma_C$ respectively. Individuals in $C_U$ class do not acquire recovery as recovery at chronic stage is mostly in late 50’s of age. Individuals in both acute classes progresses to corresponding chronic classes at the rate $\sigma_A$. Furthermore, individuals in $A_U$ and $A_F$ classes suffer additional disease-induced death at a rate $\delta_A$, while those in $C_F$ class also suffer additional disease-induced death at a rate $\delta_C$. And natural death occurs in all classes at a rate $\mu$.

The corresponding mathematical equations of the schematic diagram can be described by a system of ordinary differential equations.

\[
\frac{dS_U}{dt} = bN(1 - \varepsilon_{\rho} \tau_b) - b \vartheta(A_F + \phi C_F)(1 - \varepsilon_r \tau_U) - \left( \sigma_S + \varepsilon_r \tau_U + \mu \right) S_U
\]  
(1)

\[
\frac{dS_F}{dt} = \sigma_c S_U + \omega V - \frac{pc(A_F + \eta C_F)(1 - \varepsilon_r \tau_F)}{N} S_F - \left( \varepsilon_{\rho} \varepsilon_S + \mu \right) S_F
\]  
(2)

\[
\frac{dV}{dt} = bN(\varepsilon_{\rho} \tau_b) + \left( \varepsilon_r \tau_U \right) S_U + \left( \varepsilon_r \tau_F \right) S_F - (\omega + \mu) V
\]  
(3)

\[
\frac{dA_U}{dt} = b \vartheta(A_F + \phi C_F)(1 - \varepsilon_r \tau_b) - (\sigma_A + \mu + \delta_A) A_U
\]  
(4)

\[
\frac{dA_F}{dt} = \frac{pc(A_F + \eta C_F)(1 - \varepsilon_r \tau_F)}{N} S_F - (\sigma_A + \mu + \delta_A) A_F
\]  
(5)

\[
\frac{dC_U}{dt} = \sigma_A \varphi_U A_U - (\sigma_C + \mu) C_U
\]  
(6)

\[
\frac{dC_F}{dt} = \sigma_A \varphi_F A_F + \sigma_C C_U - (\gamma_C + \mu + \delta_C) C_F
\]  
(7)

\[
\frac{dR}{dt} = \sigma_A (1 - \varphi_U) A_U + \sigma_A (1 - \varphi_F) A_F + \gamma_C C_F - \mu R
\]  
(8)

where,

\[
N(t) = S_U(t) + S_F(t) + V(t) + A_U(t) + A_F(t) + C_U(t) + C_F(t) + R(t)
\]  
(9)

So that

\[
\frac{dN}{dt} = (b - \mu)N - \delta_A (A_U + A_F) - \delta_C C_F
\]  
(10)

in the biological - feasible region:

\[
\Omega = \left\{ (S_U, S_F, V, A_U, A_F, C_U, C_F, R) \in \mathbb{R}_+^8 : S_U + S_F + V + A_U + A_F + C_U + C_F + R = N \right\}
\]  
(11)

From the model equations (1) to (8), let

\[
\rho_b = \varepsilon_{\rho} \tau_b, \rho_U = \varepsilon_r \tau_U, \rho_F = \varepsilon_r \tau_F
\]

\[
b_A = b \vartheta(1 - \varepsilon_r \tau_b), b_C = b \vartheta \varphi(1 - \varepsilon_r \tau_U)
\]

\[
\beta = pc(1 - \varepsilon_r \tau_F), \gamma_U = \sigma_A (1 - \varphi_U), \gamma_F = \sigma_A (1 - \varphi_F)
\]  
(12)

\[
K_1 = \left( \sigma_S + \rho_U + \mu \right), K_2 = \left( \rho_F + \mu \right), K_3 = (\omega + \mu),
\]

\[
K_4 = \left( \sigma_A + \mu + \delta_A \right), K_5 = \left( \sigma_C + \mu \right), K_6 = (\gamma_C + \mu + \delta_C)
\]
Effective basic reproduction number, $R_C$

One of the most important concerns about any infectious disease is its ability to invade a population. The basic reproduction number, $R_0$, is a measure of the potential for disease spread in a population, and is inarguably 'one of the foremost and most valuable ideas that mathematical thinking has brought to epidemic theory' (Heesterbeek and Dietz, 1996). It represents the average number of secondary cases generated by an infected individual if introduced into a susceptible population with no immunity to the disease in the absence of interventions to control the infection. If $R_0 < 1$, then on average, an infected individual produces less than one newly infected individual over the course of its infection period. In this case, the infection may die out in the long run. Conversely, if $R_0 > 1$, each infected individual produces, on average more than one new infection, the infection will be able to spread in a population. A large value of $R_0$ may indicate the possibility of a major epidemic. Similarly, the effective basic reproduction number, $R_C$, represent the average number of secondary cases generated by an infected individual if introduced into a susceptible population where control strategies are used.

Using the next generation operator technique described by Heesterbeek and Dietz, (1996) and subsequently analyzed by Diekmann and Heesterbeek (2000), we obtained the effective basic reproduction number, $R_C$ of the equations (1) – (8) which is the spectral radius $\rho$ of the next generation matrix, $K$. That is, $R_C = \rho K$, where $K = FV^{-1}$

Now,

$$F = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \beta S^*_F N^* & 0 & \beta \eta S^*_F N^* \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{and} \quad V = \begin{bmatrix} K_4 & -b_A & 0 & -b_C \\ K_4 & 0 & 0 & 0 \\ -\sigma_A \varphi_U & 0 & K_5 & 0 \\ 0 & -\sigma_A \varphi_F & -\sigma_C & K_6 \end{bmatrix}$$

Then

$$\frac{dS_U}{dt} = bN \left(1 - \rho_b\right) - \left(b_A A_F + b_C C_F\right) - K_S S_U$$

$$\frac{dS_F}{dt} = \sigma_S S_U + \alpha V - \frac{\beta(A_F + \eta C_F)}{N} S_F - K_2 S_F$$

$$\frac{dV}{dt} = bN \rho_b + \rho_V S_U + \rho_F S_F - K_3 V$$

$$\frac{dA_U}{dt} = \left(b_A A_F + b_C C_F\right) - K_4 A_U$$

$$\frac{dA_F}{dt} = \frac{\beta(A_F + \eta C_F)}{N} S_F - K_4 A_F$$

$$\frac{dC_U}{dt} = \sigma_A \varphi_U A_U - K_5 C_U$$

$$\frac{dC_F}{dt} = \sigma_A \varphi_F A_F + \sigma_C C_U - K_6 C_F$$

$$\frac{dR}{dt} = \gamma_U A_U + \gamma_F A_F + \gamma_C C_F - \mu R$$

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Then
\[ R_C = \frac{b \left[ \left( \omega + \mu \right) \sigma_s + \omega \rho_b \right] \left( 1 - \rho_b \right) + \left( \sigma_s + \rho_U + \mu \right) \omega \rho_b \right]}{\mu \left( \sigma_s + \rho_U + \mu \right) \left( \omega + \rho_F + \mu \right) \left( \sigma_A + \mu + \delta_A \right)} \times \left( 1 + \frac{\eta \sigma_A \left( \left( \sigma_A + \mu + \delta_A \right) \left( \sigma_C + \mu \right) \varphi_F + \varphi_c \varphi_U \right)}{\left( \sigma_A + \mu + \delta_A \right) \left( \sigma_C + \mu \right) \left( \gamma_c + \mu + \delta_c \right) - \sigma_c \sigma_A \varphi_U \varphi_c} \right) \]

\[ (21) \]

**Results and discussions**

**Data estimation**

Model variables and parameters value usually have to be estimated based on HBV epidemiological published data and the demographic profile of the population (country) concern. As this work is concern with Nigeria, we set out in Table 1 and Table 2 the baseline values for the variables and parameters of our model respectively as described with reasons in Van den Driessche and Watmough (2002). We used 3 decimal places value accuracy for all parameters.

**Table 1:** Baseline values for variables of the HBV model in Nigeria as at year 2010

<table>
<thead>
<tr>
<th>S/N</th>
<th>Variable</th>
<th>Total Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( S_U )</td>
<td>23,148,265</td>
</tr>
<tr>
<td>2</td>
<td>( S_F )</td>
<td>51,967,535</td>
</tr>
<tr>
<td>3</td>
<td>( V )</td>
<td>16,528,817</td>
</tr>
<tr>
<td>4</td>
<td>( A_U )</td>
<td>3,012,259</td>
</tr>
<tr>
<td>5</td>
<td>( A_F )</td>
<td>5,280,601</td>
</tr>
<tr>
<td>6</td>
<td>( C_U )</td>
<td>5,394,338</td>
</tr>
<tr>
<td>7</td>
<td>( C_F )</td>
<td>6,315,313</td>
</tr>
<tr>
<td>8</td>
<td>( R )</td>
<td>40,570,172</td>
</tr>
<tr>
<td>9</td>
<td>( N )</td>
<td>152,217,300</td>
</tr>
</tbody>
</table>

**Table 2:** Baseline values for parameters of the HBV model

<table>
<thead>
<tr>
<th>S/N</th>
<th>Parameter</th>
<th>Value</th>
<th>S/N</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( b )</td>
<td>0.036</td>
<td>11</td>
<td>( \varphi_U )</td>
<td>0.885</td>
</tr>
<tr>
<td>2</td>
<td>( \mu )</td>
<td>0.021</td>
<td>12</td>
<td>( \varphi_F )</td>
<td>0.1</td>
</tr>
<tr>
<td>3</td>
<td>( c )</td>
<td>20</td>
<td>13</td>
<td>( \gamma_c )</td>
<td>0.015</td>
</tr>
<tr>
<td>4</td>
<td>( p )</td>
<td>0.079</td>
<td>14</td>
<td>( \delta_A )</td>
<td>0.007</td>
</tr>
<tr>
<td>5</td>
<td>( \eta )</td>
<td>0.667</td>
<td>15</td>
<td>( \delta_C )</td>
<td>0.001</td>
</tr>
<tr>
<td>6</td>
<td>( \theta )</td>
<td>0.724</td>
<td>16</td>
<td>( \varepsilon_{\rho} )</td>
<td>0.9</td>
</tr>
<tr>
<td>7</td>
<td>( \phi )</td>
<td>0.159</td>
<td>17</td>
<td>( \omega )</td>
<td>0.04</td>
</tr>
<tr>
<td>8</td>
<td>( \sigma_S )</td>
<td>0.067</td>
<td>18</td>
<td>( \varepsilon_{c} )</td>
<td>0.8</td>
</tr>
<tr>
<td>9</td>
<td>( \sigma_A )</td>
<td>2.667</td>
<td>19</td>
<td>( \tau_b, \tau_U, \tau_F, \tau_c )</td>
<td>(0-1)</td>
</tr>
<tr>
<td>10</td>
<td>( \sigma_C )</td>
<td>0.068</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
Computation of effective basic reproduction number, $R_C$

Using baseline values for variables and parameters as in Table 1 and Table 2 we obtained our basic reproduction number, $R_0$ for Nigeria as 5.002. As some control measures are presently in place in the country, we estimated the effective basic reproduction number, $R_C$ at present control strategy as 3.397. This is obtained using a 66% vaccination coverage on new births and assuming condom usage of 20%, vaccination coverage of 0.1% each for sexually active and yet to be sexually active susceptible individuals.

Next, we compute the effective basic reproduction number, $R_C$ of the model using the control parameters $\tau_c, \tau_b, \tau_d$ and $\tau_F$ at 6 different control strategies and rates of usage/coverage as shown in Table 3. The 6 control strategies are:

(a) Condom usage only,
(b) Vaccination of new births only,
(c) Vaccination of susceptible individuals that are not yet sexually active only,
(d) Vaccination of sexually active susceptible individuals only,
(e) Condom usage and Vaccination of new births only, and
(f) Universal strategy. Condom usage, Vaccination of new birth and susceptible individuals.

Table 3: Effect of 6 different control strategies on the effective basic reproduction number, $R_C$

<table>
<thead>
<tr>
<th>Rate of usage/coverage</th>
<th>$a$</th>
<th>$b$</th>
<th>$c$</th>
<th>$d$</th>
<th>$e$</th>
<th>$f$</th>
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</thead>
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<tr>
<td>0.1</td>
<td>4.602</td>
<td>4.940</td>
<td>4.630</td>
<td>2.008</td>
<td>4.545</td>
<td>1.699</td>
</tr>
<tr>
<td>0.2</td>
<td>4.202</td>
<td>4.877</td>
<td>4.494</td>
<td>1.251</td>
<td>4.097</td>
<td>0.935</td>
</tr>
<tr>
<td>0.3</td>
<td>3.802</td>
<td>4.815</td>
<td>4.416</td>
<td>0.905</td>
<td>3.659</td>
<td>0.601</td>
</tr>
<tr>
<td>0.4</td>
<td>3.402</td>
<td>4.752</td>
<td>4.361</td>
<td>0.707</td>
<td>3.232</td>
<td>0.415</td>
</tr>
<tr>
<td>0.5</td>
<td>3.001</td>
<td>4.690</td>
<td>4.318</td>
<td>0.579</td>
<td>2.814</td>
<td>0.297</td>
</tr>
<tr>
<td>0.6</td>
<td>2.601</td>
<td>4.628</td>
<td>4.281</td>
<td>0.490</td>
<td>2.406</td>
<td>0.215</td>
</tr>
<tr>
<td>0.7</td>
<td>2.201</td>
<td>4.565</td>
<td>4.249</td>
<td>0.423</td>
<td>2.009</td>
<td>0.156</td>
</tr>
<tr>
<td>0.8</td>
<td>1.801</td>
<td>4.503</td>
<td>4.220</td>
<td>0.372</td>
<td>1.621</td>
<td>0.112</td>
</tr>
<tr>
<td>0.9</td>
<td>1.401</td>
<td>4.440</td>
<td>4.192</td>
<td>0.332</td>
<td>1.243</td>
<td>0.077</td>
</tr>
<tr>
<td>1</td>
<td>1.000</td>
<td>4.378</td>
<td>4.166</td>
<td>0.299</td>
<td>0.876</td>
<td>0.049</td>
</tr>
</tbody>
</table>

As the effective basic reproduction number, $R_C$ is a threshold for measuring disease spread in a population, we observed from Table 3 that vaccination at birth only or before been sexually active only at any rate of coverage cannot bring about disease control as $R_C$ is greater than 1 in all cases. Though, condom usage only can lead to disease control but only 100% usage. This may not be feasible in a country like Nigeria were both cultural and religious practices/beliefs may hinder the use of condom. Vaccination of sexually active susceptible individuals, $S_F$ at any rate reduces the effective basic reproduction number, $R_C$ below unity. This is clearly an indication that 30% vaccination coverage of $S_F$’s (feasible in Nigeria) will surely lead the country toward disease free equilibrium within a short time. Although, condom usage has been practice in Nigeria long ago while vaccination at birth since 2004, positive results has not been achieved. This is because 12% were infected in 2006, while 13.14% in 2010. Table 3 clearly gives us reasons for the lack of significant impact, as almost a 100% (97% to be specific) rate of both condom usage and vaccination at birth can bring $R_C$ to less than unity, which will lead to disease control. This is not possible to attain in Nigeria as earlier discussed about condom usage. Nevertheless, the universal strategy, in
which all the four control strategies are used even for 20% usage/coverage will surely bring about disease control in short time, though as earlier pointed out the vaccination of new births should not be reduce if not for cost constrain.

**Numerical simulations**

Next, we used numerical simulations to further confirm and extend the results earlier obtained as well as provide two feasible solutions to the endemic problem of HBV in Nigeria.

![Figure 1: Comparison between the effects of present Nigeria strategy, two feasible solution and absence of any intervention on total number of chronic infection](image)

Clearly, we observed from Figure 1 that the present strategy of controlling HBV transmission in the country is better than none, though it does not have much impact on disease morbidity. Two suggested feasible solutions as illustrated in Figure 2 are:

i. Maintaining of 66% yearly vaccination coverage of new births combined with yearly 20% assumed condom usage and 10% vaccination coverage each for susceptible individual that are sexually active and yet to be sexually active.

ii. Yearly 20% assumed condom usage combined with yearly 30% vaccination coverage of susceptible individual that are sexually active.

The first can be achieved with more public enlightenment on the efficacy of the vaccine and danger of HBV. This will lead to more susceptible individuals been immunised. As the effects of the two suggested feasible solutions on chronic infection morbidity are almost the same, the latter can be employ when there is limited vaccine that will not be adequate for the population.

We now consider mortality number due to both chronic and acute infection with respect to the present strategy in the country, the two suggested feasible solution and in case all controls are stopped.
As explained earlier for Figure 1, we observed clearly in Figure 2 a drastic reduction in total mortality number due to acute infection with any of the two suggested feasible solutions. And the present strategy does not have any positive effect on the mortality number of acutely infected individuals.

Similarly, as observed in Figure 2 there is a drastic reduction in mortality number due to chronic infection with any of the two suggested feasible solutions. The second feasible solution is clear evidence that 30% vaccination coverage of sexually active susceptible individuals is enough to control chronic infection that leads to cirrhosis and HCC, as against 66% vaccination coverage at birth of the present control strategy in the country.
Figure 4: Comparison between the effects of low and high vaccinations of new births only and vaccination of sexually active susceptible individuals, \( S_F \) only on morbidity of chronically infected individuals under 15 years of age, \( C_U \).

Clearly, we observed from Figure 4 that both low (25%) and high (75%) level of vaccination coverage at birth or of \( S_F \) have highly positive effect on morbidity of \( C_U \). But 25% vaccination coverage of \( S_F \) has more impact on morbidity of \( C_U \) than 75% vaccination coverage of new births. Though, this may be against our intuitive reasoning, but for the fact that infection of the \( S_F \)’s is the main cause of the \( C_U \)’s infection. Furthermore, the recommendation of encouraging of new births vaccination more [30] is not necessary the best way to reduce HBV chronic infection of children.

Figure 5: Comparison between the effects of low and high vaccinations of new births only and vaccinations of sexually active susceptible individuals only on morbidity of chronically infected individuals at or above 15 years of age.

Unlike illustrated in Figure 4 were any level of both new births and \( S_F \) vaccination coverage reduces the morbidity number of \( C_U \), we observed no any significant impact of vaccination of new births on \( C_F \) in Figure 5. This clearly shows that vaccination of new births have significant impact on \( C_U \) only, while \( S_F \) vaccination have significant impact on both \( C_U \) and \( C_F \).
Conclusion

We presented a new mathematical model for hepatitis B virus (HBV) transmission dynamics in Nigeria, incorporating vertical transmission, public enlightenment, condom usage, standard incidence function, disease induced death due to both acute and chronic infection, vaccination and sexual maturity. We obtained the effective basic reproduction number, $R_E$, and compute its values using different control strategies. The control parameters are condom usage, $\tau_c$, and vaccination coverage at birth ($\tau_u$), susceptible individuals before sexual maturity ($\tau_U$) and when sexually active ($\tau_F$).

Our analysis reveals that with a 25 years waning rate of vaccine, the present strategy of HBV infection control can never bring about disease control in Nigeria. More effort has to be in place so that at least 20% of sexually active susceptible individuals in the country will be vaccinated yearly in addition to the 66% yearly vaccination coverage of children at birth. And if for any constraint then 30% of the sexually active susceptible individuals in the country should be vaccinated yearly.

As about 10% (4 million) of those infected with HIV are co-infected with HBV [8], there is need to mathematically analyse the co-infection in a population with vital dynamics. And finally, there is also need to look into HBV infection with regard to the immune system.

References


